King Fahd University of Petroleum and Minerals

College of Computer Science and Engineering

Information and Computer Science Department

ICS 253: Discrete Structures I

Summer Semester 2012-2013

Final Exam, Sunday July 28, 2013.

Name:

ID#:

**Instructions**:

1. This exam consists of **ten** pages, including this page and the final reference sheet, containing **five** questions.
2. You have to answer all **five** questions.
3. The exam is closed book and closed notes. Non-programmable calculators are allowed. Make sure you turn off your mobile phone and keep it in your pocket if you have one.
4. The questions are **not** **equally weighed**.
5. This exam is out of **100** points.
6. You have exactly **150** minutes to finish the exam.
7. Make sure your answers are **readable**.
8. If there is no space on the front of the page, feel free to use the back of the page. Make sure you indicate this in order for me not to miss grading it.

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| --- | --- | --- |
| Question Number | Maximum # of Points | Earned/Deducted Points |
| **I** | **20** |  |
| **II** | **20** |  |
| **III** | **25** |  |
| **IV** | **20** |  |
| **V** | **15** |  |
| **Total** | **100** |  |

1. (20 points) Choose the correct answer from the following choices. Note that the number of choices is not equal for all questions.
	1. The converse of “it rains today only if I drive to work” is
		1. if I drive to work, then it rains today.
		2. if it rains today, then I drive to work.
		3. if I don’t drive to work, then it rains today.
		4. if it does not rain today, then I drive to work.
		5. none of the above.
	2. If *F*(*p*)is “Printer *p* is out of service,” *B*(*p*)is “Printer *p* is busy,” *L*(*j*)is “Print job *j* is lost,” and *Q*(*j*)is “Print job *j* is queued.”

Then, is equivalent to

* + 1. There exists a job for all printers such that if the printer is working or is not busy, then the job is either lost or queued.
		2. For each printer that is working or is not busy, there is a job that is either lost or queued.
		3. All printers that are out of service or busy have all their jobs either lost or queued.
		4. It is not the case that there exists a printer such that if the printer is out of service or busy, all its jobs are both not lost and not queued.
		5. (b) and (d).
	1. Suppose that variable *x* represents people, and that *F*(*x*) is “*x* is friendly,” *T*(*x*) is “*x* is tall” and *A*(*x*) is “*x* is angry.” The statement “Tall angry people are not friendly” is equivalent to
		1. none of the above
	2. Which of the following statements is a true statement
		1. (*A*  *C*)  (*B*  *C*)  *A*  *B*.
		2. .
		3. If *A*  *C*  *B*  *C*, then *A*  *B*.
		4. If *A*  *C*  *B*  *C*, then *A*  *B*.
		5. none of the above.
		6. .
		7. [-1,1].
		8. 0
		9. .
		10. none of the above.
	3. =
		1. (– 1,1).
		2. [-1,1].
		3. (– 2,2).
		4. [0,1].
		5. none of the above.
	4. Suppose *f*  **N**  **N** has the rule *f*(*n*)  3*n2*  1. Then,
		1. *f* is one to one but not onto.
		2. *f* is onto but not one to one.
		3. *f* is neither one to one nor onto.
		4. *f* is one to one and onto.
		5. *f* is not a function.
	5. The minimum number of bytes needed to encode 900 bits of data equals
		1. 
		2. 
		3. 112.
		4. .
		5. More than 113.
	6. The sequence {*an*} that is a solution of the recurrence relation

 *a*n = 8*an*−1 − 16*an*−2 is

* + 1. *an* = *n*2 4*n*
		2. *an* =(−4)*n*
		3. *an* = 1
		4. *an* =2*n*
		5. 2 · 4*n* + 3*n*4*n*.
	1. =
		1. 12920425.
		2. 12920424.
		3. 505000.
		4. 338349
		5. none of the above.
1. (20 points) Induction and Recursion
	1. (10 points) For which nonnegative integers *n* is 2*n* + 3 ≤ 2*n*? Prove your answer.
	2. (10 points) Which amounts of money can be formed using just two riyal

bills and five-riyal bills? Prove your answer using strong induction.

1. (25 points) Counting
	1. (5 points) How many license plates can be made using either three uppercase English letters followed by four digits or four uppercase English letters followed by three digits?
	2. (5 points) How many ways are there to seat six people around a circular table where two seatings are considered the same when everyone has the same two neighbors without regard to whether they are right or left neighbors?
	3. (5 points) Every student in a discrete mathematics class is either a computer science or a mathematics major or is a joint major in these two subjects. How many students are in the class if there are 38 computer science majors (including joint majors), 23 mathematics majors (including joint majors), and 7 joint majors?
	4. (5 points) Find the least number of cables required to connect 100 computers to 20 printers to guarantee that every subset of 20 computers can directly access 20 different printers. Justify your answer.
	5. (5 points) Give a formula for the coefficient of *xk* in the expansion of

. Show the details of your answer.

1. (20 points) Discrete Probability
	1. (5 points) What is the probability that a five-card poker hand contains at least one ace?
	2. (5 points) In a super lottery, a player selects 5 numbers out of the first 100 positive integers. What is the probability that a person wins the grand prize by picking 5 numbers that are among the 8 numbers selected at random by a computer?
	3. (5 points) What is the conditional probability that a randomly generated bit string of length four contains at least two consecutive 0s, given that the first bit is a 1? (Assume the probabilities of a 0 and a 1 are the same.)
	4. (5 points) A coin is biased so that the probability of heads is 1/4. What is the probability that at least 8 heads come up when the coin is flipped ten times, assuming that the flips are independent?
2. (15 points) Advanced Counting Techniques
	1. (7 points) A string that contains only 0s, 1s, and 2s is called a *ternary string*.
		1. (5 points) Find a recurrence relation for the number of ternary strings of length *n* that do not contain two consecutive 0s.
		2. (2 points) What are the initial conditions?
	2. (8 points) Solve the following recurrence relation together with the initial conditions given.

*an* = 5*an* – 1 – 6*an* – 2 for *n* ≥ 2, *a*0 = 1 and *a*1 = 0

**Some Useful Formulas**

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|  | Addition |  | Modus Tollens |
|  | Simplification |  | Hypothetical syllogism |
|  | Conjunction |  | Disjunctive syllogism |
|  | Modus Ponens |  | Resolution |

